

LITERATURE CITED

1. I. F. Pikus and N. A. Gudko, *Izv. Akad. Nauk BSSR, Ser. Fiz. Energ. Nauk*, No. 2 (1978).
2. I. F. Pikus and N. A. Gudko, *Inventor's Certificate No. 593260, Byull. Izobret.*, No. 6 (1978).
3. N. A. Gudko, I. F. Pikus, et al., *Inventor's Certificate No. 658611. Byull. Izobret.*, No. 15 (1979).
4. Yu. I. Dmitrenko et al., *Elektrotekh. Prom. Elektrotekh. Mater.*, No. 10 (1977).
5. N. A. Andrianov and V. V. Skipetrov, *Synthetic Liquid Dielectrics [in Russian], Gosénergoizdat, Moscow-Leningrad* (1962).
6. I. F. Pikus, N. A. Gudko, A. N. Tarasov, and Ya. S. Raptunovich, *Elektrotekh. Prom. Ser. VVA*, No. 3 (1977).
7. I. F. Pikus, N. A. Gudko, A. G. Voskhodov, and S. I. Zhukov, *Elektrotekh. Prom. Ser. VVA*, No. 8 (1977).

NONEQUILIBRIUM THERMODYNAMICS OF
MONODISPERSE SUSPENSIONS

G. P. Yasnikov

UDC 536.70+532.135

The internal-energy and entropy balance equations of monodisperse suspensions are averaged over the statistical ensemble of possible spatial configurations of solid spherical particles.

The present work is based on the method of statistical averaging of the balance equations, valid for a liquid and at the level of individual particles, developed in [1, 2].

For the local physical quantities $G(t, \vec{r}, C_N)$ appearing in these equations, which depend on hydrodynamic - \vec{r} - and phase - $C_N(\vec{r}^{(1)}, \dots, \vec{r}^{(N)})$ - variables (N is the number of particles), the average over the distribution function $\Phi(t/C_N)$ is introduced.

The commutation properties of the averaging operator constructed in this way allow equations describing the behavior (on average) of continua which model the phases of a suspension to be obtained.

The account below is based on the matrix formalism introduced in [3]. The explicit form of the matrices and the operations involving them which are used in the present work are given in the Appendix. As in [3], consideration is restricted to a mixture of a liquid with solid spherical particles without diffusion, chemical reactions, or phase transitions. The phase materials are assumed to be incompressible and rotation of the particles insignificant.

"Microscopic" balance equations for the mechanical and total energy valid in the liquid and inside the particles may be written in the form [4]

$$D \frac{d}{dt} \left(\frac{1}{2} V^2 + \psi \right) = - \vec{\nabla} \cdot (\Sigma \cdot \vec{V}) + \Sigma : \vec{\nabla} \vec{V}, \quad (1)$$

$$D \frac{de}{dt} = - \vec{\nabla} \cdot (\Sigma \cdot \vec{V}) - \vec{\nabla} \cdot \vec{J}_q. \quad (2)$$

After multiplying Eqs. (1) and (2) by Θ - Eq. (A.1) - and averaging, the following relations are obtained:

$$\langle \Theta D \frac{d}{dt} \left(\frac{1}{2} V^2 + \psi \right) \rangle = - \langle \Theta \vec{\nabla} \cdot (\Sigma \cdot \vec{V}) \rangle + \langle \Sigma : (\vec{\nabla} \vec{V}) \Theta \rangle, \quad (3)$$

$$\langle \Theta D \frac{de}{dt} \rangle = - \langle \Theta \vec{\nabla} \cdot (\Sigma \cdot \vec{V}) \rangle - \langle \Theta \vec{\nabla} \cdot \vec{J}_q \rangle. \quad (4)$$

Term-by-term subtraction in Eqs. (2) and (1) and in Eqs. (4) and (3) gives the internal-energy balance equations

$$D \frac{du}{dt} = -\Sigma : \vec{\nabla} \vec{V} - \vec{\nabla} \cdot \vec{J}_q, \quad (5)$$

$$\langle \Theta D \frac{du}{dt} \rangle = -\langle \Theta \Sigma : \vec{\nabla} \vec{V} \rangle - \langle \Theta \vec{\nabla} \cdot \vec{J}_q \rangle. \quad (6)$$

The terms appearing on the right-hand sides of Eqs. (3), (4), and (6) may be transferred using Eqs. (A.2)-(A.6), $G = \Sigma, \vec{V}, \vec{J}_q$,

$$\begin{aligned} \langle \Theta \vec{\nabla} \cdot \Sigma \cdot \vec{V} \rangle &= a \vec{\nabla} \cdot \text{Sp} [(\Theta \sigma \cdot \mathbf{v}) + \langle \Sigma' : \vec{V}' \rangle] + (b-a) \langle \Theta_1 (\vec{\nabla} \cdot \Sigma) \rangle \cdot \mathbf{v} + \\ &+ (b-a) \langle \Theta_1 (\vec{\nabla} \cdot \Sigma) \vec{V}' \rangle + (b-a) \langle \Theta_1 \Sigma : \vec{\nabla} \vec{V} \rangle = a \vec{\nabla} \cdot [\text{Sp} (\Theta \sigma \cdot \mathbf{v}) + \\ &+ \langle \Sigma' : \vec{V}' \rangle] + (b-a) \vec{f} \cdot \mathbf{v} + (b-a) \langle \Theta_1 (\vec{\nabla} \cdot \Sigma) \vec{V}' \rangle + (b-a) \langle \Theta_1 \Sigma : \vec{\nabla} \vec{V} \rangle, \end{aligned} \quad (7)$$

where $\vec{f} = -\langle \Theta_1 \vec{\nabla} \cdot \Sigma \rangle$ is the phase-interaction force, and

$$\langle \Theta \Sigma : \vec{\nabla} \vec{V} \rangle = a \text{Sp} (\Theta \sigma : \vec{\nabla} \mathbf{v}) + a \langle \Sigma' : \vec{\nabla} \vec{V}' \rangle + (b-a) \langle \Theta_1 \Sigma : \vec{\nabla} \vec{V} \rangle; \quad (8)$$

$$\langle \Theta \vec{\nabla} \cdot \vec{J}_q \rangle = a \vec{\nabla} \cdot \text{Sp} (\Theta \mathbf{q}) + (b-a) \langle \Theta_1 \vec{\nabla} \cdot \vec{J}_q \rangle. \quad (9)$$

The matrices σ , \mathbf{v} , and \mathbf{q} are defined in Eq. (A.7). The last terms in Eqs. (7)-(9) characterize surface effects and may be expressed in terms of integrals over the surface of the test particle. The left-hand sides of Eqs. (3), (4), and (6) may be transformed as in Eq. (A.8). Using Eqs. (7), (9), and (A.8), the total-energy balance equation - Eq. (4) - may be written in the form

$$\begin{aligned} \frac{\partial}{\partial t} (\text{De}) + \vec{\nabla} \cdot (\text{Dev} + d \langle \Theta e' \vec{V}' \rangle) &= -a \vec{\nabla} \cdot [\text{Sp} (\Theta \sigma \cdot \mathbf{v}) + \langle \Sigma' : \vec{V}' \rangle] - \\ &- a \vec{\nabla} \cdot \text{Sp} (\Theta \mathbf{q}) + (b-a) \vec{f} \cdot \mathbf{v} + (b-a) \langle \Theta_1 (\vec{\nabla} \cdot \Sigma) \rangle \cdot \mathbf{v} - (b-a) \langle \Theta_1 \Sigma : \vec{\nabla} \vec{V} \rangle - (b-a) \langle \Theta_1 \vec{\nabla} \cdot \vec{J}_q \rangle. \end{aligned} \quad (10)$$

Analogously, the mechanical-energy balance equation - Eq. (3) - may be written in the form [3]

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{1}{2} \text{Dv}^2 + \frac{1}{2} d \langle \Theta \vec{V}' \cdot \vec{V}' \rangle + \text{D}\Psi \right) + \vec{\nabla} \cdot \left[\frac{1}{2} \text{Dv}^2 \mathbf{v} + \right. \\ \left. + \frac{1}{2} d \mathbf{v} \langle \Theta \vec{V}' \cdot \vec{V}' \rangle + \frac{1}{2} d \langle \Theta \vec{V}' (\vec{V}' \cdot \vec{V}') \rangle + \text{Dv}\Psi + d \langle \Theta \psi \vec{V}' \rangle + \right. \end{aligned} \quad (11)$$

$$\left. + \sigma^* \cdot \mathbf{v} \right] = -a \vec{\nabla} \cdot [\text{Sp} (\Theta \sigma \cdot \mathbf{v}) + \langle \Sigma' : \vec{V}' \rangle] + (b-a) \vec{f} \cdot \mathbf{v} + (b-a) \langle \Theta_1 (\vec{\nabla} \cdot \Sigma) \vec{V}' \rangle + a \text{Sp} (\Theta \sigma : \vec{\nabla} \mathbf{v}) + a \langle \Sigma' : \vec{\nabla} \vec{V}' \rangle.$$

The terms on the right-hand sides of Eqs. (10) and (11) on which the operator $\vec{\nabla}$ acts represent the fluxes of mechanical and thermal energy, while the other terms act as the corresponding sources and are equal to the power of the forces acting in unit volume of the suspension. The matrices appearing in these equations are defined in Eqs. (A.7)-(A.10).

Subtracting Eq. (11) term by term from Eq. (10) and taking into account that $s = V^2/2 + \psi + u$, the averaged internal-energy balance equation is obtained. The same result may be obtained if the left-hand side of Eq. (6) is rewritten using Eq. (A.8), and Eqs. (8) and (9) are used in the right-hand side:

$$\begin{aligned} \frac{\partial}{\partial t} (\text{Du}) + \vec{\nabla} \cdot (\text{Duv} + d \langle \Theta u' \vec{V}' \rangle) &= -a \vec{\nabla} \cdot \text{Sp} (\Theta \mathbf{q}) - a \text{Sp} (\Theta \sigma : \vec{\nabla} \mathbf{v}) - \\ &- a \langle \Theta_1 \Sigma : \vec{\nabla} \vec{V} \rangle + (b-a) \langle \Theta_1 \vec{\nabla} \cdot \vec{J}_q \rangle - (b-a) \langle \Theta_1 \Sigma : \vec{\nabla} \vec{V} \rangle. \end{aligned} \quad (12)$$

It is evident from Eqs. (6) and (12) that the internal energy changes as a result of heat fluxes and the power developed by internal stress but, in contrast to the equation for a single-phase medium, Eq. (12) includes terms associated with surface effects.

The averaged entropy balance equation is derived using the "microscopic" balance equation and the fundamental Gibbs equation,

$$D \frac{ds}{dt} = -\vec{\nabla} \cdot \vec{J}_s + \sigma_s, \quad (13)$$

$$DT \frac{ds}{dt} = D \frac{du}{dt} + \Sigma^{(e)} : \vec{\nabla} \vec{V}, \quad (14)$$

where $\Sigma^{(e)}$ is the reversible part of the pressure tensor.

After multiplying Eqs. (13) and (14) by Θ and averaging, the following relations are obtained:

$$\langle \Theta D \frac{ds}{dt} \rangle = - \langle \Theta \vec{\nabla} \cdot \vec{J}_s \rangle + \langle \Theta \sigma_s \rangle, \quad (15)$$

$$\langle \Theta D \frac{ds}{dt} \rangle = \langle \Theta DT^{-1} \frac{du}{dt} \rangle + \langle \Theta T^{-1} \Sigma^{(e)} : \vec{\nabla} \vec{V} \rangle. \quad (16)$$

The first term on the right-hand side of Eq. (16) may be expressed using Eq. (5). This involves multiplying Eq. (5) by ΘT^{-1} and averaging, with the result

$$\langle \Theta D \frac{ds}{dt} \rangle = - \langle \Theta T^{-1} \Sigma^{(i)} : \vec{\nabla} \vec{V} \rangle - \langle \Theta T^{-1} \vec{\nabla} \cdot \vec{J}_q \rangle. \quad (17)$$

Here $\Sigma^{(i)} = \Sigma - \Sigma^{(e)}$ is the irreversible part of the pressure tensor.

Using Eqs. (A.2), (A.3), and (A.8), Eq. (15) is transformed to give

$$\frac{\partial}{\partial t} (\mathbf{Ds}) + \vec{\nabla} \cdot (\mathbf{Dsv} + \mathbf{d} \langle \Theta s' \vec{V}' \rangle) = - \mathbf{a} \vec{\nabla} \cdot \text{Sp}(\Theta \mathbf{J}_s) + (\mathbf{b} - \mathbf{a}) \langle \theta_1 \vec{\nabla} \cdot \vec{J}_s \rangle + \mathbf{a} \text{Sp}(\Theta \sigma_s) + (\mathbf{b} - \mathbf{a}) \langle \theta_1 \sigma_s \rangle. \quad (18)$$

The matrices $\mathbf{J}_s \sigma_s$ and \mathbf{s} are given in Eq. (A.11).

The first term in Eq. (17) is transformed using Eqs. (A.2) and (A.6) ($G = \Sigma^{(i)}, \vec{\nabla}, T^{-1}$) and the approximate relations

$$IT^{-1} = (\tau + IT')^{-1} \cong \tau^{-1} - \tau^{-2} T' + \dots, \quad (19)$$

$$\begin{aligned} \langle \Theta T^{-1} \Sigma^{(i)} : \vec{\nabla} \vec{V} \rangle &\cong \mathbf{a} \text{Sp}(\tau^{-1} \Theta \sigma^{(i)} : \vec{\nabla} \mathbf{v}) + \mathbf{a} \text{Sp} \tau^{-1} \langle \Theta \Sigma^{(i)'} : \vec{\nabla} \vec{V}' \rangle + \\ &+ (\mathbf{b} - \mathbf{a}) \langle \Theta T^{-1} \Sigma^{(i)} : \vec{\nabla} \vec{V} \rangle - \tau^{-2} \langle \Theta T' \Sigma^{(i)} : \vec{\nabla} \vec{V} \rangle + \dots, \end{aligned} \quad (20)$$

and the second term in Eq. (16) is written in the form

$$\begin{aligned} \langle \Theta T^{-1} \vec{\nabla} \cdot \vec{J}_q \rangle &= \langle \Theta \vec{\nabla} \cdot (\vec{J}_q T^{-1}) \rangle - \langle \Theta \vec{J}_q \cdot \vec{\nabla} T^{-1} \rangle = \mathbf{a} \vec{\nabla} \cdot \text{Sp}(\Theta \mathbf{q} \tau^{-1}) + \\ &+ \mathbf{a} \text{Sp}(\Theta \mathbf{q} \tau^{-2} \cdot \vec{\nabla} \tau) + (\mathbf{b} - \mathbf{a}) \langle \theta_1 \vec{\nabla} \cdot (\vec{J}_q T^{-1}) \rangle - (\mathbf{b} - \mathbf{a}) \langle \theta_1 \vec{J}_q \cdot \vec{\nabla} T^{-1} \rangle + \\ &+ \mathbf{a} \text{Sp}(\mathbf{q} \tau^{-2} \cdot \langle \Theta \vec{\nabla} T' \rangle) + \mathbf{a} \vec{\nabla} \text{Sp}(\tau^{-2} \langle \Theta \vec{J}_q' T' \rangle) + \mathbf{a} \langle \Theta \vec{J}_q' \cdot \vec{\nabla} (\tau^{-2} T') \rangle + \dots \end{aligned} \quad (21)$$

If Eqs. (20) and (21) are substituted into Eq. (17), the left-hand side of Eq. (17) is transformed using Eq. (A.8), and terms of the order of the square of the pulsation divided by the square of the mean temperature are neglected, then the entropy balance equation is obtained in the form

$$\begin{aligned} \frac{\partial}{\partial t} (\mathbf{Ds}) + \vec{\nabla} \cdot (\mathbf{Dsv} + \mathbf{d} \langle \Theta s' \vec{V}' \rangle) &= - \mathbf{a} \vec{\nabla} \cdot \text{Sp}(\Theta \tau^{-1} \mathbf{q}) - \\ &- (\mathbf{b} - \mathbf{a}) \langle \Theta_1 \vec{\nabla} \cdot (T^{-1} \vec{J}_q) \rangle - \mathbf{a} \text{Sp}(\Theta \mathbf{q} \tau^{-2} \cdot \vec{\nabla} \tau) - \mathbf{a} \text{Sp}(\mathbf{q} \tau^{-2} \cdot \langle \Theta \vec{\nabla} T' \rangle) + \\ &+ (\mathbf{b} - \mathbf{a}) \langle \theta_1 \vec{J}_q \cdot \vec{\nabla} T^{-1} \rangle - \mathbf{a} \text{Sp}(\tau^{-1} \Theta \sigma^{(i)} : \vec{\nabla} \mathbf{v}) + \mathbf{a} \text{Sp}(\tau^{-1} \langle \Theta \Sigma^{(i)'} : \vec{\nabla} \vec{V}' \rangle) - (\mathbf{b} - \mathbf{a}) \langle \Theta T' \Sigma^{(i)} : \vec{\nabla} \vec{V} \rangle. \end{aligned} \quad (22)$$

The matrices $\mathbf{s}, \tau, \tau^{-1}, \mathbf{q}$, and $\sigma^{(i)}$ are given in Eqs. (A.11) and (A.12). If the terms on the right-hand sides of Eqs. (22) and (18) are compared, their physical meaning becomes clear. The first two terms

$$\vec{\nabla} I_a = - \mathbf{a} \vec{\nabla} \cdot \text{Sp}(\Theta \mathbf{J}_s) - (\mathbf{b} - \mathbf{a}) \langle \theta_1 \vec{\nabla} \cdot \vec{J}_s \rangle = \mathbf{a} \vec{\nabla} \cdot \text{Sp}(\Theta \mathbf{q} \tau^{-1}) - (\mathbf{b} - \mathbf{a}) \langle \theta_1 \vec{\nabla} \cdot (T^{-1} \vec{J}_q) \rangle \quad (23)$$

define the energy fluxes (per unit volume of mixture) due to heat transfer in the mean temperature fields of the liquid and solid phases and at the particle surface (in the latter the pulsational components may be explicitly separated). The remaining terms constitute the contribution to the energy production of irreversible heat transfer and viscous friction, which appears in the mean and pulsational motion of liquids perturbed by particles and at the particle surface.

$$\begin{aligned}
\sigma_a = & a \text{Sp}(\theta\sigma_s) + (b-a) \langle \theta_s \sigma_s \rangle = -a \text{Sp}(\theta q \tau^{-2} \cdot \vec{\nabla} \tau) - \\
& -a \text{Sp}(q \tau^{-2} \cdot \langle \theta \vec{\nabla} T' \rangle) - a \text{Sp}(\tau^{-1} \theta \sigma^{(i)} : \vec{\nabla} \mathbf{v}) + a \text{Sp}(\tau^{-1} \langle \theta \Sigma^{(i)'} : \vec{\nabla} \vec{V}' \rangle) + \\
& + (b-a) \langle \theta_i \vec{J}_q \cdot \vec{\nabla} T^{-1} \rangle + (b-a) \langle \theta T^{-1} \Sigma^{(i)} : \vec{\nabla} \vec{V}' \rangle.
\end{aligned} \tag{24}$$

Formally introducing the effective entropy flux I_a and the effective energy production σ_a of the suspension, the entropy balance equation takes the same form in Eq. (13) as for a single-phase medium,

$$\frac{\partial}{\partial t} (Ds) + \vec{\nabla} \cdot (Dsv + d \langle \theta s' \vec{V}' \rangle) = -\vec{\nabla} \cdot I_a + \sigma_a.$$

The entropy balance for the suspension phases is written in the usual form: for the liquid

$$\begin{aligned}
\frac{\partial}{\partial t} (e d_0 s_0 \vec{v}_0 + d_0 \langle \theta_0 s' \vec{V}' \rangle) = & -\vec{\nabla} \cdot (e \vec{q}_0 \tau_0^{-1} + \rho \vec{q}_1 \tau_1^{-1}) - \langle \theta_i \vec{\nabla} \cdot (\vec{J}_q \cdot T^{-1}) \rangle - \\
& - (e \vec{q}_0 \cdot \tau_0^{-2} \vec{\nabla} \tau_0 + \rho \vec{q}_1 \cdot \tau_1^{-2} \vec{\nabla} \tau_1) - (e \vec{q}_0 \cdot \tau_0^{-2} \langle \theta_0 \vec{\nabla} T' \rangle + \rho \vec{q}_1 \cdot \tau_1^{-2} \langle \theta_i \vec{\nabla} T' \rangle) + \\
& + \langle \theta_i \vec{J}_q \cdot T^{-2} \vec{\nabla} T \rangle - (\tau_0^{-1} e \sigma_0^{(i)} : \vec{\nabla} \vec{v}_0 + \tau_1^{-1} \rho \sigma_1^{(i)} : \vec{\nabla} \vec{v}_1) - \\
& - (\tau_0^{-1} \langle \theta_0 \Sigma^{(i)'} : \vec{\nabla} \vec{V}' + \tau_1^{-1} \langle \theta_i \Sigma^{(i)'} : \vec{\nabla} \vec{V}' \rangle) + \langle \theta_i T^{-1} \Sigma^{(i)} : \vec{\nabla} \vec{V}' \rangle,
\end{aligned} \tag{25}$$

for the particles

$$\frac{\partial}{\partial t} (\rho d_1 s_1) + \vec{\nabla} \cdot (\rho d_1 s_1 \vec{v}_1 + d_1 \langle \theta_1 s' \vec{V}' \rangle) = \langle \theta_i \vec{J}_q \cdot T^{-2} \vec{\nabla} T \rangle + \langle \theta_i \vec{\nabla} \cdot (\vec{J}_q T^{-1}) \rangle - \langle \theta_i T^{-1} \Sigma^{(i)} : \vec{\nabla} \vec{V}' \rangle. \tag{26}$$

Adding together Eqs. (25) and (26) gives the entropy balance equation for the mixture. To establish the mutual influence of the irreversible processes, it is necessary to know the symmetry properties of the medium [4]. According to the Curie theorem, vector and tensor phenomena do not interfere in an isotropic medium. In this case, for example, the heat-conduction law in the medium may be written phenomenologically on the basis of Eq. (24) in the form

$$\vec{q} = e \vec{q}_0 + \rho \vec{q}_1 = -\varepsilon \frac{L_0}{\tau_0^2} \vec{\nabla} (\tau_0 + \langle \theta_0 \vec{\nabla} T' \rangle) - \rho \frac{L_1}{\tau_1^2} \vec{\nabla} (\tau_1 + \langle \theta_i \vec{\nabla} T' \rangle). \tag{27}$$

Here $L_0 = \lambda_0 \tau_0^2$, $L_1 = \lambda_1 \tau_1^2$ are phenomenological coefficients; λ_0 and λ_1 are the thermal conductivities of the phases. The same relation may be obtained by formal averaging of the "microscopic" Fourier phenomenological law

$$\vec{J}_q = -\Lambda \vec{\nabla} T, \tag{28}$$

where $\Lambda = \theta_0 \lambda_0 + \theta_i \lambda_1$ is the generalized thermal conductivity. On the other hand, the averaged value of the heat flux in Eq. (28) may be written in the form

$$\vec{q} = \langle \vec{J}_q \rangle = -\lambda_0 \vec{\nabla} (e \tau_0 + \rho \tau_1) - (\lambda_1 - \lambda_0) \langle \theta_i \vec{\nabla} T' \rangle, \tag{29}$$

completely equivalent to Eq. (27). Equation (29) has already been derived and used by Buevich in considering heat conduction in suspensions.*

Analogous consideration of viscous dissipation allows the effective stress tensor to be written as follows [1]:

$$\sigma = \langle \Sigma \rangle = -\langle \Sigma^{(e)} + \Sigma^{(i)} \rangle = -\varepsilon \rho_0 I + 2\mu_0 [\vec{\nabla} \cdot \text{Sp} \theta \mathbf{v}]^s.$$

The superscript s denotes the symmetric part of the mixture's strain-rate tensor.

The equations considered in the present work and in [1-3] completely describe the thermal and hydrodynamic behavior of suspensions of liquid with solid particles. It remains of interest to include diffusion and chemical reactions in the given scheme, and to take rotation of the particles into account.

*Yu. A. Buevich, Paper at the Fifth All-Union Conference on Heat and Mass Transfer [in Russian], ITMO, Minsk (1976).

APPENDIX

$$\Theta = \begin{pmatrix} \theta_0 & 0 \\ 0 & \theta_1 \end{pmatrix}, \quad \text{Sp } \Theta = 1, \quad \theta_1 = \sum_i^N \eta (a - |\vec{r} - \vec{r}^{(i)}|). \quad (\text{A.1})$$

For the generalized function C with the mean $\langle \Theta G \rangle$, the following formulas are valid [3]:

$$\begin{aligned} \langle \Theta G \rangle &= \Theta \mathbf{g}, \quad \langle G \rangle = \mathbf{g} = \text{Sp } \langle \Theta G \rangle = \text{Sp } \Theta \mathbf{g}, \\ \langle \Theta G \rangle &= a \text{Sp } (\Theta \mathbf{g}) + (b - a) \langle \theta_1 G \rangle, \end{aligned} \quad (\text{A.2})$$

$$\langle \Theta \vec{\nabla} G \rangle = a \vec{\nabla} \text{Sp } (\Theta \mathbf{g}) + (b - a) \langle \theta_1 \vec{\nabla} G \rangle, \quad (\text{A.3})$$

$$\Theta = \langle \Theta \rangle = \begin{pmatrix} \varepsilon & 0 \\ 0 & \rho \end{pmatrix}, \quad \mathbf{g} = \begin{pmatrix} g_0 & 0 \\ 0 & g_1 \end{pmatrix}, \quad a + b = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (\text{A.4})$$

For the generalized function

$$G = \Sigma, \quad \vec{V}, \vec{J}_q, \psi, e, u, s, \dots \quad (\text{A.5})$$

the pulsational components

$$IG = \mathbf{g} + IG', \quad (\text{A.6})$$

$$\mathbf{g} = \sigma = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} \vec{v}_0 & 0 \\ 0 & \vec{v}_1 \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} \vec{q}_0 & 0 \\ 0 & \vec{q}_1 \end{pmatrix} \quad (\text{A.7})$$

may be introduced.

The averaged value of the substantial derivative dG/dt is [3]

$$\langle D\Theta \frac{dG}{dt} \rangle = \frac{\partial}{\partial t} (\mathbf{D}\mathbf{g}) + \vec{\nabla} \cdot (\mathbf{D}\mathbf{g}\mathbf{v} + \mathbf{d} \langle \Theta \vec{V}' \vec{V}' \rangle), \quad (\text{A.8})$$

$$D = \theta_0 d_0 + \theta_1 d_1, \quad \mathbf{D} = \begin{pmatrix} \varepsilon d_0 & 0 \\ 0 & \rho d_1 \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} d_0 & 0 \\ 0 & d_1 \end{pmatrix}, \quad (\text{A.9})$$

$$\mathbf{e} = \begin{pmatrix} e_0 & 0 \\ 0 & e_1 \end{pmatrix}, \quad \Psi = \begin{pmatrix} \psi_0 & 0 \\ 0 & \psi_1 \end{pmatrix}, \quad \sigma^* = \mathbf{d} \langle \Theta \vec{V}' \vec{V}' \rangle, \quad (\text{A.10})$$

$$\mathbf{s} = \begin{pmatrix} s_0 & 0 \\ 0 & s_1 \end{pmatrix}, \quad \mathbf{J}_s = \begin{pmatrix} \vec{J}_{s0} & 0 \\ 0 & \vec{J}_{s1} \end{pmatrix}, \quad \sigma_s = \begin{pmatrix} \sigma_{s0} & 0 \\ 0 & \sigma_{s1} \end{pmatrix}, \quad (\text{A.11})$$

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_0 & 0 \\ 0 & \tau_1 \end{pmatrix}, \quad \boldsymbol{\tau}^{-1} = \begin{pmatrix} \tau_0^{-1} & 0 \\ 0 & \tau_1^{-1} \end{pmatrix}, \quad \sigma^{(i)} = \begin{pmatrix} \sigma_0^{(i)} & 0 \\ 0 & \sigma_1^{(i)} \end{pmatrix}. \quad (\text{A.12})$$

NOTATION

Local quantities:

\vec{V}	is the velocity;
ψ	is the potential energy;
$\Sigma(i)$	is the viscous-pressure tensor;
$\Sigma(e)$	is the reversible part of pressure tensor;
Σ	is the complete pressure tensor;
\vec{J}_q	is the heat-flux density;
θ_0, θ_1	are the generalized functions defined in the Appendix;
s	is the specific entropy;
u	is the specific internal energy;
Λ	is the generalized thermal conductivity;

T	is the temperature;
\vec{J}_s	is the entropy flux density;
σ_s	is the entropy production;
e	is the specific total energy.

Matrices with elements obtained on averaging local quantities:

β	is the porosity;
\mathbf{v}	is the velocity;
σ	is the pressure tensor;
\mathbf{q}	are the heat fluxes;
D	is the generalized density;
e	is the total energy;
τ	is the temperature;
s	is the entropy;
\mathbf{J}_s	is the entropy flux;
σ_s	is the entropy production;
σ^*	is the effective pulsational stress;
$\vec{\sigma}^{(i)}$	is the viscous stress;
\vec{f}	is the mean phase-interaction force;
d_0, d_1	are the phase-material densities;
L_0, L_1	are the phenomenological coefficients;
ε	is the porosity;
P_0	is the liquid pressure;
μ_0	is the liquid viscosity;
η	is the Heaviside function;
a	is the particle radius.

Subscripts:

0	liquid;
1	particles.

Symbols:

$\langle \dots \rangle$	is the averaging over an ensemble;
(\cdot)	is the scalar product;
$(:)$	is the doubly convoluted tensor-element product;

A bar above a symbol denotes the pulsational component.

LITERATURE CITED

1. Yu. A. Buevich and I. N. Shchelchkova, Continuum Mechanics of Monodisperse Suspensions [in Russian], Preprint No. 72, IMP Akad. Nauk SSSR, Moscow (1978).
2. Yu. A. Buevich (Buyevich) and I. N. Shchelchkova, Prog. Aerospace Sci., 18, No. 2A (1978).
3. G. P. Yasnikov, Inzh. -Fiz. Zh., 37, No. 4 (1979).
4. S. R. De Groot and P. Mazur, Non-Equilibrium Thermodynamics, Elsevier (1962).